UUCMS. No. | | | | | | | | | | | |

B.M.S. COLLEGE FOR WOMEN, AUTONOMOUS

BENGALURU – 560004

SEMESTER END EXAMINATION – SEPT/OCT 2023

M.Sc. in Mathematics – 2nd Semester

NUMERICAL ANALYSIS –I

Course Code: MM205T **QP Code: 12005**

Duration: 3 Hours **Max. Marks: 70**

Instructions: ***1) All questions carry equal marks.***
2) Answer any five full questions.

1. (a) Find the root of $2x - \cos(x) - 3 = 0$ with $x_0 = \frac{\pi}{2}$ correct to four decimal places using Aitken's Δ^2 method.
(b) Prove or disprove that the Newton-Raphson method for finding a simple root of the equation $f(x) = 0$ has quadratic convergence whereas linearly for finding a multiple root.
(6+8)

2. (a) Find the smallest root of the equation $xe^x = 1$ using Ramanujan's method.
(b) Extract a quadratic factor of the form $px^2 + qx + 1 = 0$ from $x^4 - x^3 + 6x^2 + 5x + 10 = 0$ using Bairstow method. Take $(p_0, q_0) = (1.14, 1.42)$. **(7+7)**

3. (a) Solve the following system of equations using Gauss elimination method.
$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z - 16$$
(b) Explain the LU decomposition method for solving a system of algebraic equations $AX = B$.
(7+7)

4. (a) Establish the Gauss-Seidel iteration method for solving a system of algebraic equations $AX = B$ in the matrix form.
(b) Solve the following system of equations using homotopy continuation method.
$$y\cos(xy) + 1 = 0$$
$$\sin(xy) + x - y = 0$$
Take $(x_0, y_0) = (1,2)$ as the initial approximation. **(7+7)**

5. (a) Find the Lagrange interpolating polynomial that fits the following data. Also find an approximation to $f(x)$ at $x = 3$ using the polynomial.

x	0	1	2	4
$f(x)$	2	5	12	62

(b) Find the error in representing a function by a Hermite interpolating polynomial when (x_i, y_i, y_i') are given. **(7+7)**

6. (a) Find the least squares approximation of second degree that fits the following data.

x	-2	-1	0	1	2
$f(x)$	15	11	1	3	19

(b) Find the rational approximation $R_{2,3}$ for $\cos(\sqrt{x})$. **(7+7)**

7. (a) Derive the Newton-Cotes methods and hence deduce the trapezoidal and Simpson's rule.

(b) Evaluate $\int_{-1}^{1}(1-x^2)^{\frac{3}{2}}\cos(x)\,dx$ using Gauss-Chebyshev two- and three-point quadrature formulae. **(9+5)**

8. (a) Establish Gauss-Hermite two- and three-point quadrature formulae. Hence evaluate $\int_{-\infty}^{\infty}\frac{e^{-x^2}}{1+x^2}dx$.

(b) Evaluate $\int_0^1\int_0^2\frac{2xy}{(1+x^2)(1+y^2)}dy\,dx$ using trapezoidal rule with $h = k = 0.5$.

(8+6)

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